On the symmetry of universal finite-size scaling functions in anisotropic systems

2002 J. Phys. A: Math. Gen. 35 L481

(http://iopscience.iop.org/0305-4470/35/31/103)

J. Phys. A: Math. Gen. **35** (2002) L481–L487 PII: S0305-4470(02)37167-1

LETTER TO THE EDITOR

On the symmetry of universal finite-size scaling functions in anisotropic systems

Alfred Hucht

Theoretische Physik, Gerhard-Mercator-Universität, D-47048 Duisburg, Germany

E-mail: fred@thp.Uni-Duisburg.de

Received 20 May 2002
Published 26 July 2002
Online at stacks.iop.org/JPhysA/35/L481

Abstract
We postulate a symmetry of universal finite-size scaling functions under a certain anisotropic scale transformation, which connects the properties of a finite two-dimensional (2D) system at criticality with generalized aspect ratio $\rho > 1$ to a system with $\rho < 1$. The symmetry is formulated within a finite-size scaling theory and expressions for several universal amplitude ratios are derived. The predictions are confirmed within the exactly solvable weakly anisotropic 2D Ising model and are checked within the strongly anisotropic 2D dipolar in-plane Ising model using Monte Carlo simulations.

PACS numbers: 05.70.Fh, 75.40.Cx, 05.50.+q

The theory of universal finite-size scaling (UFSS) functions is a key concept in the modern understanding of continuous phase transitions [1–3]. In particular, it is known that the UFSS functions of a rectangular two-dimensional (2D) system of size $L_\| \times L_\perp$ depend on the aspect ratio $L_\|/L_\perp$ [4]. For instance, in *isotropic* systems, the scaling function at criticality $\bar{U}_\mathrm{c}$ of the Binder cumulant $U = 1 - \frac{1}{3}\langle m^4\rangle/\langle m^2\rangle^2$ [5], where $\langle m^n\rangle$ is the nth moment of the order parameter, is known to be a universal function $\bar{U}_\mathrm{c}(L_\|/L_\perp)$ for a given boundary condition. This quantity has been investigated by several authors in the isotropic 2D Ising model with periodic boundary conditions [6, 7], while the influence of other boundary conditions on $\bar{U}_\mathrm{c}(L_\|/L_\perp)$ has recently been studied in [8, 9].

In *weakly anisotropic* systems, where the couplings are anisotropic ($J_\| \neq J_\perp$ in the 2D Ising case), the correlation length of the infinite system in direction $\mu = \|, \perp$ becomes anisotropic and scales like $\xi_\mu^{(\infty)}(t) \sim \hat{\xi}_\mu t^{-\nu}$ near criticality. ($t = (T - T_\mathrm{c})/T_\mathrm{c}$ is the reduced temperature and we assume $t > 0$, without loss of generality.) This leads to a correlation length amplitude ratio $\hat{\xi}_\|/\hat{\xi}_\perp$ different from unity. The UFSS functions then depend on this ratio, i.e. $\bar{U}_\mathrm{c} = \bar{U}_\mathrm{c}(L_\|/L_\perp, \hat{\xi}_\|/\hat{\xi}_\perp)$. However, isotropy can be restored asymptotically by an anisotropic scale transformation, where all lengths are rescaled with the corresponding correlation length amplitudes $\hat{\xi}_\mu$ [10–12]. Thus, the UFSS functions depend on $L_\|/L_\perp$ and $\hat{\xi}_\|/\hat{\xi}_\perp$ only through the *reduced* aspect ratio $(L_\|/\hat{\xi}_\|)/(L_\perp/\hat{\xi}_\perp)$.

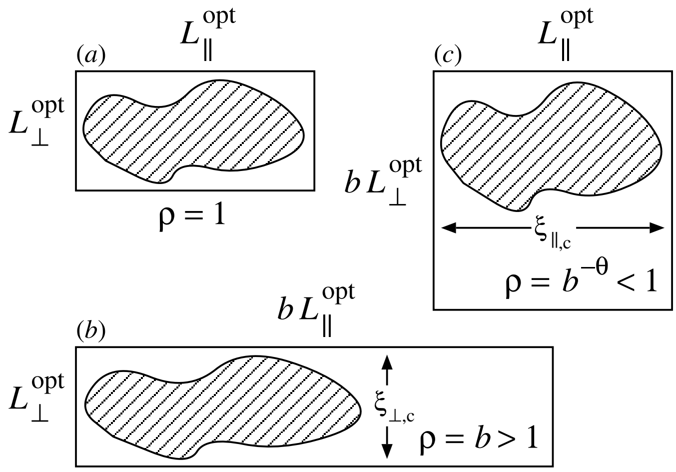

Figure 1. Three systems with different aspect ratio ρ (equation (2)) at criticality. In (*a*), the critical correlation volume $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}}$ (shaded area) spans the whole system, while in (*b*) and (*c*), correlations are limited by symmetric finite-size effects.

In *strongly anisotropic* systems, both the amplitudes $\hat{\xi}_\mu$ as well as the correlation length exponents ν_μ are different and the correlation length in direction μ scales like

$$\xi_\mu^{(\infty)}(t) \sim \hat{\xi}_\mu t^{-\nu_\mu}. \tag{1}$$

Examples for strongly anisotropic phase transitions are Lifshitz points [13] as present in the anisotropic next nearest neighbour Ising (ANNNI) model [14–16], or the non-equilibrium phase transition in the driven lattice gas model [17, 18]. Furthermore, in dynamical systems one can identify the $\parallel$-direction with time and the $\perp$-direction(s) with space [19], which in most cases give strongly anisotropic behaviour.

Using the same arguments as above, we conclude that UFSS functions of strongly anisotropic systems depend on the *generalized* reduced aspect ratio (cf [6])

$$\rho = L_\parallel L_\perp^{-\theta} \big/ r_\xi \qquad \text{with} \quad r_\xi = \hat{\xi}_\parallel \hat{\xi}_\perp^{-\theta} \tag{2}$$

being the *generalized* correlation length amplitude ratio, and with the anisotropy exponent $\theta = \nu_\parallel/\nu_\perp$ [19]. Up to now, no attempts have been made to describe the dependence of UFSS functions such as $\bar{U}_\mathrm{c}(\rho)$ on the shape ρ of strongly anisotropic systems. In particular, it is not known if the anisotropy exponent θ can be determined from $\bar{U}_\mathrm{c}(\rho)$. This problem is addressed in this work.

Consider a 2D strongly anisotropic finite system with periodic boundary conditions. When the critical point of the infinite system is approached from temperatures $t > 0$, the correlation lengths ξ_μ in the different directions μ are limited by the direction in which $\xi_\mu^{(\infty)}$ from equation (1) reaches the system boundary first [4]. For a given volume $N = L_\parallel L_\perp$, we define an 'optimal' shape $L_\parallel^{\mathrm{opt}} \times L_\perp^{\mathrm{opt}}$ at which both correlation lengths $\xi_\mu^{(\infty)}$ reach the system boundary simultaneously, i.e.

$$L_\mu^{\mathrm{opt}} := \xi_\mu^{(\infty)}(t) \tag{3}$$

for some temperature $t > 0$ (figure 1(*a*)). We immediately find, using equations (1) and (2), that the optimal shape obeys $\rho_{\mathrm{opt}} \equiv 1$ for all N, giving $L_\parallel^{\mathrm{opt}} = r_\xi (L_\perp^{\mathrm{opt}})^\theta$. A system of optimal shape should show the strongest critical fluctuations for a given volume N as the critical correlation volume $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}}$ spans the whole system.

At the optimal aspect ratio $\rho = 1$, the correlations are limited by both directions $\parallel$ and $\perp$ (figure 1(*a*)). If the system is enlarged by a factor $b > 1$ in the $\parallel$-direction (figure 1(*b*)), the correlation volume may relax into this direction but does not fill the whole system due to the limitation in the $\perp$-direction. A similar situation with exchanged roles occurs if the system is enlarged by a factor $b > 1$ in the $\perp$-direction (figure 1(*c*)). We now *assume* that systems (*b*) and (*c*) are similar in the scaling region $L_\mu^{\text{opt}} \to \infty$, i.e. their correlation volumes are asymptotically equal.

Hence, we can formulate a *symmetry hypothesis*: Consider a system with periodic boundary conditions and optimal aspect ratio $\rho = 1$ at the critical point. If this system is enlarged by a factor $b > 1$ in the $\parallel$-direction, it behaves asymptotically the same as if enlarged by *the same factor* b in the $\perp$-direction.

To formulate this hypothesis within a finite-size scaling theory, we consider a 2D strongly anisotropic system of size $L_\parallel \times L_\perp$ which fulfils the generalized hyperscaling relation $2 - \alpha = \nu_\parallel + \nu_\perp$ [6]. For our purpose, it is sufficient to focus on the critical point. The universal finite-size scaling *ansatz* [1–4, 6] for the singular part of the free energy density $f_c = F_{s,c}/(Nk_B T_c)$ reads [20]

$$f_c(L_\parallel, L_\perp) \sim \frac{b_\parallel b_\perp}{N} Y_c(b_\parallel, b_\perp) \tag{4}$$

with the scaling variables $b_\mu = \lambda^{\nu_\mu} L_\mu / \hat{\xi}_\mu$, where λ is a free-scaling parameter. The scaling function Y_c is universal for a given boundary condition, and all non-universal properties are contained in the metric factors $\hat{\xi}_\mu$. These metric factors occur due to the usual requirement that the relevant lengths are $L_\mu / \xi_\mu^{(\infty)}(t)$ near criticality and cannot be absorbed into λ in contrast to isotropic systems. For the three systems in figure 1, we set $\lambda = \left(L_\mu^{\text{opt}} \big/ \hat{\xi}_\mu\right)^{-1/\nu_\mu}$ to get

$$f_c\left(L_\parallel^{\text{opt}}, L_\perp^{\text{opt}}\right) \sim \frac{1}{N} Y_c(1, 1) \tag{5a}$$

$$f_c\left(bL_\parallel^{\text{opt}}, L_\perp^{\text{opt}}\right) \sim \frac{b}{N} Y_c(b, 1) \tag{5b}$$

$$f_c\left(L_\parallel^{\text{opt}}, bL_\perp^{\text{opt}}\right) \sim \frac{b}{N} Y_c(1, b). \tag{5c}$$

The proposed symmetry hypothesis states that for $b > 1$, equations (5*b*) and (5*c*) are asymptotically equal in the scaling region where L_μ^{opt} is large,

$$f_c\left(bL_\parallel^{\text{opt}}, L_\perp^{\text{opt}}\right) \overset{b>1}{\simeq} f_c\left(L_\parallel^{\text{opt}}, bL_\perp^{\text{opt}}\right). \tag{6}$$

Hence, the scaling function Y_c has the simple symmetry

$$Y_c(b, 1) \overset{b>1}{=} Y_c(1, b). \tag{7}$$

To rewrite Y_c as a function of the generalized aspect ratio ρ (equation (2)), instead of the quantities b_μ, we set $b_\perp = 1$ in system (*c*) and get, as then $\lambda = \left(bL_\perp^{\text{opt}}/\hat{\xi}_\perp\right)^{-1/\nu_\perp}$,

$$f_c\left(L_\parallel^{\text{opt}}, bL_\perp^{\text{opt}}\right) \sim \frac{b^{-\theta}}{N} Y_c(b^{-\theta}, 1). \tag{8}$$

Equations (5*c*) and (8) are identical and we conclude that $bY_c(1, b) = b^{-\theta} Y_c(b^{-\theta}, 1)$. At this point, it is convenient to define the scaling function $\bar{Y}_c(b) = bY_c(b, 1)$ which fulfils

$$f_c(L_\parallel, L_\perp) \sim \frac{1}{N} \bar{Y}_c(\rho). \tag{9}$$

For this scaling function, the symmetry reads

$$\bar{Y}_{\rm c}(\rho) \stackrel{\rho\geq 1}{=} \bar{Y}_{\rm c}(\rho^{-\theta}). \tag{10}$$

We see from equation (9) that the critical free energy density $f_{\rm c}$ is a universal function of the reduced aspect ratio $\rho = L_\| L_\perp^{-\theta} \big/ r_\xi$ without any non-universal prefactor, and that at criticality, *all* system specific properties are contained in the non-universal ratio r_ξ from equation (2).

Ansatz equation (4) can also be made for the inverse spin–spin correlation length at criticality [20]

$$\xi_{\mu,{\rm c}}^{-1}(L_\|, L_\perp) \sim \frac{b_\mu}{L_\mu} X_{\mu,{\rm c}}(b_\|, b_\perp). \tag{11}$$

The proposed symmetry gives $X_{\mu,{\rm c}}(b,1) \stackrel{b\geq 1}{=} X_{\bar{\mu},{\rm c}}(1,b)$, where $\bar{\mu}$ denotes the direction perpendicular to μ. Hence, the correlation volumes $\xi_{\|,{\rm c}}\xi_{\perp,{\rm c}}$ of systems (*b*) and (*c*) in figure 1 are indeed equal as assumed above and become $\xi_{\|,{\rm c}}\xi_{\perp,{\rm c}} \sim \frac{N}{b} X_{\|,{\rm c}}^{-1}(b,1) X_{\perp,{\rm c}}^{-1}(b,1)$.

The correlation length amplitudes A_ξ^μ in cylindrical geometry ($b_\mu \to \infty, b_{\bar{\mu}} = 1$), which can be calculated exactly for many isotropic two-dimensional models within the theory of conformal invariance [21], generalize to the strongly anisotropic form [3]

$$A_\xi^\mu = \lim_{L_{\bar{\mu}}\to\infty} L_{\bar{\mu}}^{-\nu_\mu/\nu_{\bar{\mu}}} \lim_{L_\mu\to\infty} \xi_{\mu,{\rm c}}(L_\|, L_\perp). \tag{12}$$

Inserting equation (11), they become

$$A_\xi^\| = r_\xi X_{\|,{\rm c}}^{-1}(\infty, 1) \qquad A_\xi^\perp = r_\xi^{-1/\theta} X_{\perp,{\rm c}}^{-1}(1,\infty) \tag{13}$$

which shows that in general A_ξ^μ is not universal. The symmetry hypothesis states that both limits of the scaling function $X_{\mu,{\rm c}}$ are equal. Denoting this universal limit by $A_\xi := X_{\|,{\rm c}}^{-1}(\infty,1) = X_{\perp,{\rm c}}^{-1}(1,\infty)$, we obtain $A_\xi^\| = r_\xi A_\xi$ and $A_\xi^\perp = r_\xi^{-1/\theta} A_\xi$ as well as the amplitude relations

$$A_\xi^{1+\theta} = A_\xi^\| \left(A_\xi^\perp\right)^\theta \qquad \frac{A_\xi^\|}{A_\xi^\perp} = r_\xi^{1+1/\theta}. \tag{14}$$

These predictions can be checked within the exactly solved weakly anisotropic 2D Ising model with different couplings $J_\|$ and $J_\perp$, where the paramagnetic correlation length reads $\xi_\mu^{(\infty)}(t) = (\log\coth(\beta J_\mu) - 2\beta J_{\bar{\mu}})^{-1}$ with $\beta = 1/k_{\rm B}T$ [22]. The amplitude ratio r_ξ at the critical point $\sinh(2\beta_{\rm c}J_\|)\sinh(2\beta_{\rm c}J_\perp) = 1$ [22] becomes $r_\xi = \sinh(2\beta_{\rm c}J_\|)$ [23]. On the other hand, the inverse correlation length amplitudes in cylinder geometry, equation (12), have been calculated [24] to give $A_\xi^\mu = \frac{4}{\pi}\sinh(2\beta_{\rm c}J_\mu)$, which immediately yields equations (13) if we insert the well-known universal value $A_\xi = 4/\pi$ [21, 25]. The left relation of equations (14) has already been derived for several weakly anisotropic models, where it simplifies to $A_\xi^2 = A_\xi^\| A_\xi^\perp$ [24, equation (7)].

To check the symmetry numerically in strongly anisotropic systems, we now focus on the Binder cumulant U. The scaling *ansatz* at criticality equation (4) becomes

$$U_{\rm c}(L_\|, L_\perp) \sim \frac{1}{b_\| b_\perp} \tilde{U}_{\rm c}(b_\|, b_\perp) = \bar{U}_{\rm c}(\rho) \tag{15}$$

with the scaling function $\bar{U}_{\rm c}(b) = \tilde{U}_{\rm c}(b,1)/b$, and the calculation is completely analogous to the free-energy case. The symmetry hypothesis for the cumulant scaling functions $\tilde{U}_{\rm c}$ and $\bar{U}_{\rm c}$ thus reads (cf equations (7) and (10))

$$\tilde{U}_{\rm c}(b,1) \stackrel{b\geq 1}{=} \tilde{U}_{\rm c}(1,b) \qquad \bar{U}_{\rm c}(\rho) \stackrel{\rho\geq 1}{=} \bar{U}_{\rm c}(\rho^{-\theta}). \tag{16}$$

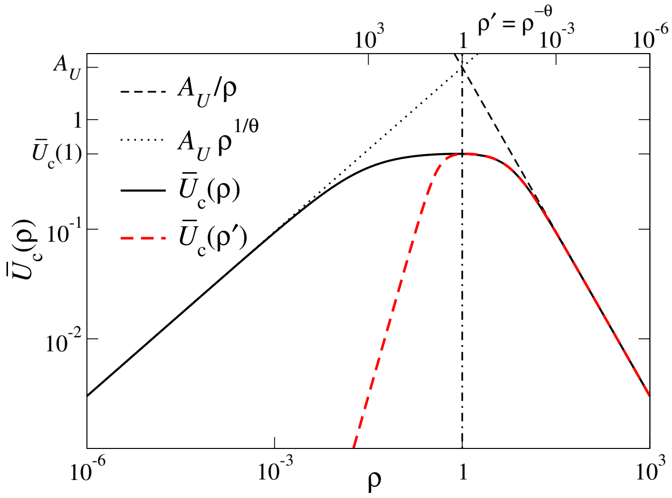

Figure 2. Sketch of critical cumulant scaling functions $\bar{U}_{\mathrm{c}}(\rho)$ and $\bar{U}_{\mathrm{c}}(\rho')$ with $\rho' = \rho^{-\theta}$ for assumed anisotropy exponent $\theta = 2$. We have $\bar{U}_{\mathrm{c}}(\rho \gg 1) \sim A_U/\rho$ and $\bar{U}_{\mathrm{c}}(\rho \ll 1) \sim A_U \rho^{1/\theta}$. For $\rho > 1$ $\bar{U}_{\mathrm{c}}(\rho)$ fulfils $\bar{U}_{\mathrm{c}}(\rho) = \bar{U}_{\mathrm{c}}(\rho')$.

The generalization of the cumulant amplitude A_U^μ [5, 26] to strongly anisotropic systems is similar to equation (12) and gives

$$A_U^\mu = \lim_{L_{\bar{\mu}} \to \infty} L_{\bar{\mu}}^{-\nu_\mu/\nu_{\bar{\mu}}} \lim_{L_\mu \to \infty} L_\mu U_{\mathrm{c}}(L_\parallel, L_\perp). \tag{17}$$

Inserting the scaling *ansatz* equation (15) we now find

$$A_U^\parallel = r_\xi \tilde{U}_{\mathrm{c}}(\infty, 1) \qquad A_U^\perp = r_\xi^{-1/\theta} \tilde{U}_{\mathrm{c}}(1, \infty) \tag{18}$$

which again are, in general, not universal. Using the symmetry hypothesis, we can define $A_U := \tilde{U}_{\mathrm{c}}(\infty, 1) = \tilde{U}_{\mathrm{c}}(1, \infty)$ and get $A_U^\parallel = r_\xi A_U$, $A_U^\perp = r_\xi^{-1/\theta} A_U$ as well as the identities (cf equations (14))

$$A_U^{1+\theta} = A_U^\parallel \left(A_U^\perp\right)^\theta \qquad \frac{A_U^\parallel}{A_U^\perp} = r_\xi^{1+1/\theta}. \tag{19}$$

The cumulant scaling function $\bar{U}_{\mathrm{c}}(\rho)$ must be extremal at $\rho = 1$ due to symmetry. Furthermore, as a deviation from the optimal aspect ratio $\rho = 1$ reduces the cumulant, it has a maximum at this point [6]. A sketch of $\bar{U}_{\mathrm{c}}(\rho)$ for an assumed anisotropy exponent $\theta = 2$ is depicted in figure 2. For $\rho > 1$, both $\bar{U}_{\mathrm{c}}(\rho)$ and $\bar{U}_{\mathrm{c}}(\rho' = \rho^{-\theta})$ collapse onto a single curve, reflecting the proposed symmetry. It is obvious from figure 2 that $\bar{U}_{\mathrm{c}}(\rho)$ (and thus, also $\bar{Y}_{\mathrm{c}}(\rho)$ from equation (10)) cannot be analytic at $\rho = 1$ in strongly anisotropic systems, as the two branches $\bar{U}_{\mathrm{c}}(\rho)$ and $\bar{U}_{\mathrm{c}}(\rho')$ identical for $\rho > 1$ fork at $\rho = 1$ [20]. On the other hand, $\bar{Y}_{\mathrm{c}}(\rho)$ and $\bar{U}_{\mathrm{c}}(\rho)$ can be analytic at $\rho = 1$ if the anisotropy exponent $\theta = 1$, as in the case of the isotropic 2D Ising model [27, equation (3.37)].

To check the symmetry hypothesis in a strongly anisotropic system, I performed Monte Carlo simulations of the two-dimensional dipolar in-plane Ising model [20]

$$\mathcal{H} = -\frac{J}{2} \sum_{\langle ij \rangle} \sigma_i \sigma_j + \frac{\omega}{2} \sum_{i \neq j} \frac{\left(r_{ij}^\perp\right)^2 - 2\left(r_{ij}^\parallel\right)^2}{|\vec{r}_{ij}|^5} \sigma_i \sigma_j \tag{20}$$

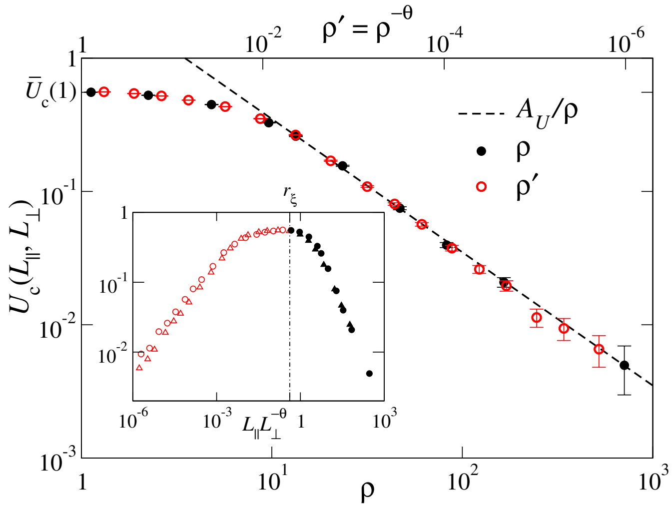

Figure 3. Cumulant $U_c(L_\parallel, L_\perp)$ of the dipolar in-plane Ising model (equation (20)) for dipole strength $\omega/J = 0.1$ and system size $N = 43\,200$ at the critical point $k_B T_c/J = 2.764(1)$. The data points collapse for $\rho > 1$ if we set $\theta = 2.1(3)$ and $r_\xi = 0.415(40)$, giving the universal amplitudes $\bar{U}_c(1) = 0.555(5)$ and $A_U = 3.5(2)$. The inset shows U_c as a function of the non-reduced generalized aspect ratio $L_\parallel L_\perp^{-\theta}$ for system size $N = 43\,200$ (circles) and $N = 4320$ (triangles).

with spin variables $\sigma = \pm 1$, ferromagnetic nearest neighbour exchange interaction $J > 0$ and dipole interaction $\omega > 0$. The distance $\vec{r}_{ij} = \left(r_{ij}^\parallel, r_{ij}^\perp\right)$ between spins σ_i and σ_j is decomposed into contributions parallel and perpendicular to the spin axis. In the simulations, the Wolff cluster algorithm [28] for long-range systems proposed by Luijten and Blöte [29] was used, modified to anisotropic interactions. In contrast to earlier work [30, 31] using renormalization group techniques, it is found that this model shows a strongly anisotropic phase transition. The details of the simulations will be published elsewhere [20].

After T_c was determined, systems with constant volume $N = L_\parallel L_\perp$ were simulated, which was chosen to have a large number of divisors in order to get many different aspect ratios (e.g., $N = 2^6 3^3 5^2 = 43\,200$ has 84 divisors). The resulting critical cumulant $U_c(L_\parallel L_\perp^{-\theta})$ for two different volumes $N = 4320, 43\,200$ is depicted in the inset of figure 3. As expected, both curves have the same maximum value $\bar{U}_c(1) = 0.555(5)$ at criticality. With variation of θ, the curves are shifted horizontally and collapse for $\theta = 2.1(3)$, with maximum at $r_\xi = 0.415(40)$. To check the proposed symmetry, we fold the left branch with $\rho < 1$ (open symbols) to the right and rescale the ρ-axis with θ. The resulting data collapse for $\rho > 1$ is shown in figure 3. This collapse and the additional condition that both curves must go to zero as A_U/ρ allows a precise determination of θ and r_ξ as well as of the universal amplitude $A_U = 3.5(2)$.

In conclusion, I postulate a symmetry of universal finite-size scaling functions under a certain anisotropic scale transformation and generalize the Privman–Fisher equations [1] to strongly anisotropic phase transitions on rectangular lattices at criticality. It turns out that for a given boundary condition, the only relevant variable is the generalized reduced aspect ratio $\rho = L_\parallel L_\perp^\theta / r_\xi$ and that, e.g., the free energy scaling function equation (9) obeys the symmetry $\bar{Y}_c(\rho) \overset{\rho \geq 1}{=} \bar{Y}_c(\rho^{-\theta})$. At criticality, the free energy density f_c, the inverse correlation lengths $\xi_{\mu,c}$ and the Binder cumulant U_c are universal functions of ρ, without a non-universal prefactor. All system specific properties are contained in the non-universal correlation length amplitude ratio r_ξ (equation (2)).

The generalization to higher dimensions is straightforward [20], an interesting application would be the precise determination of the exponent θ at the Lifshitz point of the three-dimensional ANNNI model [15, 16]. An open question is the validity of the proposed symmetry in non-equilibrium systems with appropriate boundary conditions, which have recently been shown to exhibit Privman–Fisher universality [3].

Acknowledgments

I thank Sven Lübeck and Erik Luijten for valuable discussions and Malte Henkel for a critical reading of the manuscript. This work was supported by the Deutsche Forschungsgemeinschaft through SFB 491.

References

[1] Privman V and Fisher M E 1994 *Phys. Rev.* B **30** 322
[2] Privman V 1990 *Finite Size Scaling and Numerical Simulation of Statistical Systems* ed V Privman (Singapore: World Scientific) ch 1
[3] Henkel M and Schollwöck U 2001 *J. Phys. A: Math. Gen.* **34** 3333
[4] Binder K 1990 *Finite Size Scaling and Numerical Simulation of Statistical Systems* ed V Privman (Singapore: World Scientific) ch 4
[5] Binder K 1981 *Z. Phys.* B **43** 119
[6] Binder K and Wang J-S 1989 *J. Stat. Phys.* **55** 87
[7] Kamieniarz G and Blöte H W J 1993 *J. Phys. A: Math. Gen.* **26** 201
[8] Okabe Y, Kaneda K, Kikuchi M and Hu C-K 1999 *Phys. Rev.* E **59** 1585
[9] Kaneda K and Okabe Y 2001 *Phys. Rev. Lett.* **86** 2134
[10] Landau D P and Swendsen R H 1984 *Phys. Rev.* B **30** 2787
[11] Indekeu J O, Nightingale M P and Wang W V 1986 *Phys. Rev.* B **34** 330
[12] Yurishchev M A 1997 *Phys. Rev.* E **55** 3915
[13] Hornreich R M, Luban M and Shtrikman S 1975 *Phys. Rev. Lett.* **35** 1678
[14] Selke W 1992 *Phase Transitions and Critical Phenomena* vol 15, ed C Domb and M S Green (London: Academic)
[15] Diehl H W and Shpot M 2000 *Phys. Rev.* B **62** 12338
[16] Pleimling M and Henkel M 2001 *Phys. Rev. Lett.* **87** 125702
[17] Katz S, Lebowitz J L and Spohn H 1983 *Phys. Rev.* B **28** 1655
[18] Schmittmann B and Zia R K P 1995 *Phase Transitions and Critical Phenomena* vol 17, ed C Domb and J L Lebowitz (London: Academic)
[19] Henkel M 1999 *Conformal Invariance and Critical Phenomena (Texts and Monographs in Physics)* (Berlin: Springer)
[20] Hucht A 2002 in preparation
[21] Cardy J 1984 *J. Phys. A: Math. Gen.* **17** L385
[22] Onsager L 1944 *Phys. Rev.* **65** 117
[23] Zia R K P and Avron J E 1982 *Phys. Rev.* B **25** 2042
[24] Nightingale P and Blöte H 1983 *J. Phys. A: Math. Gen.* **16** L657
[25] Luck J M 1982 *J. Phys. A: Math. Gen.* **15** L169
[26] Burkhardt T W and Derrida B 1985 *Phys. Rev.* B **32** 7273
[27] Ferdinand A E and Fisher M E 1969 *Phys. Rev.* **185** 832
[28] Wolff U 1989 *Phys. Rev. Lett.* **62** 361
[29] Luijten E and Blöte H W J 1995 *Int. J. Mod. Phys.* C **6** 359
[30] De'Bell K and Geldart D J W 1989 *Phys. Rev.* B **39** 743
[31] Bulenda M, Täuber U C and Schwabl F 2000 *J. Phys. A: Math. Gen.* **33** 1